



Teaching for Mastery

Questions, tasks and activities to support assessment

Year 4

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About the authors

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Introduction

In line with the curricula of many high performing jurisdictions, the National curriculum emphasises the importance of all pupils mastering the content taught each year and discourages the acceleration of pupils into content from subsequent years.

The current National curriculum document¹ says:

'The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.' (National curriculum page 3)

Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills. Key measures of this are the abilities to reason mathematically and to solve increasingly complex problems, doing so with fluency, as described in the aims of the National curriculum:

'The national curriculum for mathematics aims to ensure that all pupils:

- *become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately*
- ***reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language*
- *can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.'* (National curriculum page 3)



Assessment arrangements must complement the curriculum and so need to mirror these principles and offer a structure for assessing pupils' progress in developing mastery of the content laid out for each year. Schools, however, are only *'required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study'* (National curriculum page 4). Schools should identify when they will teach the programmes of study and set out their school curriculum for mathematics on a year-by-year basis. The materials in this document reflect the arrangement of content as laid out in the National curriculum document (September 2013).

These *Teaching for Mastery: Questions, tasks and activities to support assessment* outline the key mathematical skills and concepts within each yearly programme and give examples of questions, tasks and practical classroom activities which support teaching, learning and assessment. The activities offered are not intended to address each and every programme of study statement in the National curriculum. Rather, they aim to highlight the key themes and big ideas for each year.

1. Mathematics programmes of study: key stages 1 and 2, National curriculum in England, September 2013, p3



Ongoing assessment as an integral part of teaching

The questions, tasks, and activities that are offered in the materials are intended to be a useful vehicle for assessing whether pupils have mastered the mathematics taught.

However, the best forms of ongoing, formative assessment arise from well-structured classroom activities involving interaction and dialogue (between teacher and pupils, and between pupils themselves). The materials are not intended to be used as a set of written test questions which the pupils answer in silence. They are offered to indicate valuable learning activities to be used as an integral part of teaching, providing rich and meaningful assessment information concerning what pupils know, understand and can do.

The tasks and activities need not necessarily be offered to pupils in written form. They may be presented orally, using equipment and/or as part of a group activity. The encouragement of discussion, debate and the sharing of ideas and strategies will often add to both the quality of the assessment information gained and the richness of the teaching and learning situation.

What do we mean by mastery?

The essential idea behind mastery is that **all children**² need a **deep** understanding of the mathematics they are learning so that:

- future mathematical learning is built on solid foundations which do not need to be re-taught;
- there is no need for separate catch-up programmes due to some children falling behind;
- children who, under other teaching approaches, can often fall a long way behind, are better able to keep up with their peers, so that gaps in attainment are narrowed whilst the attainment of all is raised.

There are generally four ways in which the term mastery is being used in the current debate about raising standards in mathematics:

- 1. A mastery approach:** a set of principles and beliefs. This includes a belief that all pupils are capable of understanding and doing mathematics, given sufficient time. Pupils are neither 'born with the maths gene' nor 'just no good at maths'. With good teaching, appropriate resources, effort and a 'can do' attitude all children can achieve in and enjoy mathematics.
- 2. A mastery curriculum:** one set of mathematical concepts and big ideas for all. All pupils need access to these concepts and ideas and to the rich connections between them. There is no such thing as 'special needs mathematics' or 'gifted and talented mathematics'. Mathematics is mathematics and the key ideas and building blocks are important for everyone.
- 3. Teaching for mastery:** a set of pedagogic practices that keep the class working together on the same topic, whilst at the same time addressing the need for all pupils to master the curriculum and for some to gain greater depth of proficiency and understanding. Challenge is provided by going deeper rather than accelerating into new

- 2.** Schools in England are required to adhere to the 0-25 years SEND Code of Practice 2015 when considering the provision for children with special educational needs and/or disability. Some of these pupils may have particular medical conditions that prevent them from reaching national expectations and will typically have a statement of Special Educational Needs/ Education Health Care Plan. Wherever possible children with special educational needs and/or a disability should work on the same curriculum content as their peers; however, it is recognised that a few children may need to work on earlier curriculum content than that designated for their age. The principle, however, of developing deep and sustainable learning of the content they are working on should be applied.

mathematical content. Teaching is focused, rigorous and thorough, to ensure that learning is sufficiently embedded and sustainable over time. Long term gaps in learning are prevented through speedy teacher intervention. More time is spent on teaching topics to allow for the development of depth and sufficient practice to embed learning. Carefully crafted lesson design provides a scaffolded, conceptual journey through the mathematics, engaging pupils in reasoning and the development of mathematical thinking.

- 4. Achieving mastery of particular topics and areas of mathematics.** Mastery is not just being able to memorise key facts and procedures and answer test questions accurately and quickly. It involves knowing 'why' as well as knowing 'that' and knowing 'how'. It means being able to use one's knowledge appropriately, flexibly and creatively and to apply it in new and unfamiliar situations.³ The materials provided seek to exemplify the types of skills, knowledge and understanding necessary for pupils to make good and sustainable progress in mastering the primary mathematics curriculum.

Mastery and the learning journey

Mastery of mathematics is not a fixed state but a continuum. At each stage of learning, pupils should acquire and demonstrate sufficient grasp of the mathematics relevant to their year group, so that their learning is sustainable over time and can be built upon in subsequent years. This requires development of depth through looking at concepts in detail using a variety of representations and contexts and committing key facts, such as number bonds and times tables, to memory.

Mastery of facts, procedures and concepts needs time: time to explore the concept in detail and time to allow for sufficient practice to develop fluency.

3. Helen Drury asserts in 'Mastering Mathematics' (Oxford University Press, 2014, page 9) that: 'A mathematical concept or skill has been mastered when, through exploration, clarification, practice and application over time, a person can represent it in multiple ways, has the mathematical language to be able to communicate related ideas, and can think mathematically with the concept so that they can independently apply it to a totally new problem in an unfamiliar situation.'

Practice is most effective when it is intelligent practice,⁴ i.e. where *the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity.* (Gu 2004⁵) The examples provided in the materials seek to exemplify this type of practice.

Mastery and mastery with greater depth

Integral to mastery of the curriculum is the development of deep rather than superficial conceptual understanding. '*The research for the review of the National Curriculum showed that it should focus on "fewer things in greater depth", in secure learning which persists, rather than relentless, over-rapid progression.*'⁶ It is inevitable that some pupils will grasp concepts more rapidly than others and will need to be stimulated and challenged to ensure continued progression. However, research indicates that these pupils benefit more from enrichment and deepening of content, rather than acceleration into new content. Acceleration is likely to promote superficial understanding, rather than the true depth and rigour of knowledge that is a foundation for higher mathematics.⁷

Within the materials the terms *mastery* and *mastery with greater depth* are used to acknowledge that all pupils require depth in their learning, but some pupils will go deeper still in their learning and understanding.

Mastery of the curriculum requires that all pupils:

- use mathematical concepts, facts and procedures appropriately, flexibly and fluently;
- recall key number facts with speed and accuracy and use them to calculate and work out unknown facts;
- have sufficient depth of knowledge and understanding to reason and explain mathematical concepts and procedures and use them to solve a variety of problems.

4. Intelligent practice is a term used to describe practice exercises that integrate the development of fluency with the deepening of conceptual understanding. Attention is drawn to the mathematical structures and relationships to assist in the deepening of conceptual understanding, whilst at the same time developing fluency through practice.

5. Gu, L., Huang, R., & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. In Lianghuo, F., Ngai-Ying, W., Jinfa, C., & Shiqi, L. (Eds.) *How Chinese learn mathematics: Perspectives from insiders*. Singapore: World Scientific Publishing Co. Pte. Ltd. page 315.

6. Living in a Levels-Free World, Tim Oates, published by the Department for Education <https://www.tes.co.uk/teaching-resource/living-in-a-levels-free-world-by-tim-oates-6445426>

7. This argument was advanced by the Advisory Committee for Mathematics Education on page 1 of its report: Raising the bar: developing able young mathematicians, December 2012.

A useful checklist for what to look out for when assessing a pupil's understanding might be:

A pupil really understands a mathematical concept, idea or technique if he or she can:

- describe it in his or her own words;
- represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)⁸
- explain it to someone else;
- make up his or her own examples (and non-examples) of it;
- see connections between it and other facts or ideas;
- recognise it in new situations and contexts;
- make use of it in various ways, including in new situations.⁹

Developing mastery with greater depth is characterised by pupils' ability to:

- solve problems of greater complexity (i.e. where the approach is not immediately obvious), demonstrating creativity and imagination;
- independently explore and investigate mathematical contexts and structures, communicate results clearly and systematically explain and generalise the mathematics.

The materials seek to exemplify what these two categories of *mastery* and *mastery with greater depth* might look like in terms of the type of tasks and activities pupils are able to tackle successfully. It should, however, be noted that the two categories are not intended to exemplify differentiation of activities/tasks. Teaching for mastery requires that all pupils are taught together and all access the same content as exemplified in the first column of questions, tasks and activities. The questions, tasks and activities exemplified in the second column might be used as deepening tasks for pupils who grasp concepts rapidly, but can also be used with the whole class where appropriate, giving all children the opportunity to think and reason more deeply.

National curriculum assessments

National assessment at the end of Key Stages 1 and 2 aims to assess pupils' mastery of both the content of the curriculum and the depth of their understanding and application of mathematics. This is exemplified through the content and cognitive domains of the test frameworks.¹⁰ The content domain exemplifies the minimum content pupils are required to evidence in order to show mastery of the curriculum. The cognitive domain aims to measure the complexity of application and depth of pupils' understanding. The questions, tasks and activities provided in these materials seek to reflect this requirement to master content in terms of both skills and depth of understanding.

Final remarks

These resources are intended to assist teachers in teaching and assessing for mastery of the curriculum. In particular they seek to exemplify what depth looks like in terms of the types of mathematical tasks pupils are able to successfully complete and how some pupils can achieve even greater depth. A key aim is to encourage teachers to keep the class working together, spend more time on teaching topics and provide opportunities for all pupils to develop the depth and rigour they need to make secure and sustained progress over time.

8. The Concrete-Pictorial-Abstract (CPA) approach, based on Bruner's conception of the enactive, iconic and symbolic modes of representation, is a well-known instructional heuristic advocated by the Singapore Ministry of Education since the early 1980s. See <https://www.ncetm.org.uk/resources/44565> (free registration required) for an introduction to this approach.

9. Adapted from a list in 'How Children Fail', John Holt, 1964.

10. 2016 Key stage 1 and 2 Mathematics test frameworks, Standards and Assessments Agency
www.gov.uk/government/collections/national-curriculum-assessments-test-frameworks

The structure of the materials

The materials consist of PDF documents for each year group from Y1 to Y6. Each document adopts the same framework as outlined below.

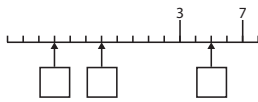
The examples provided in the materials are only indicative and are designed to provide an insight into:

- How mastery of the curriculum might be developed and assessed;
- How to teach the same curriculum content to the whole class, challenging the rapid graspers by supporting them to go deeper rather than accelerating some pupils into new content.

The assessment activities presented in both columns are suitable for use with the whole class. Pupils who successfully answer the questions in the left-hand column (Mastery) show evidence of sufficient depth of knowledge and understanding. This indicates that learning is likely to be sustainable over time. Pupils who are also successful with answering questions in the right-hand column (Mastery with Greater Depth) show evidence of greater depth of understanding and progress in learning.

This section lists a selection of key National Curriculum programme of study statements. The development and assessment of these is supported through the questions, tasks and activities set out in the two columns below.

This section lists a selection of key ideas relevant to the selected programme of study statements.

Number and Place Value	
<p>Selected National Curriculum Programme of Study Statements</p> <p>Pupils should be taught to:</p> <ul style="list-style-type: none"> ■ count in multiples of 6, 7, 9, 25 and 1000 ■ order and compare numbers beyond 1000 ■ count backwards through 0 to include negative numbers ■ round any number to the nearest 10, 100 or 1000 	
<p>The Big Ideas</p> <p>Imagining the position of numbers on a horizontal number line helps us to order them: the number to the right on a number line is the larger number. So 5 is greater than 4, as 5 is to the right of 4. But -4 is greater than -5 as -4 is to the right of -5.</p> <p>Rounding numbers in context may mean rounding up or down. Buying packets of ten cakes, we might round up to the nearest ten to make sure everyone gets a cake. Estimating the number of chairs in a room for a large number of people we might round down the number of chairs to make sure there are enough.</p> <p>We can think of place value in additive terms: 456 is $400 + 50 + 6$, or in multiplicative terms: one hundred is ten times as large as ten.</p>	
<p>Mastery Check</p> <p>Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if ...?', and checking that pupils can use the procedures or skills to solve a variety of problems.</p>	
<p>Mastery</p> <p>Write the missing numbers in the boxes.</p> 	<p>Mastery with Greater Depth</p> <p>The sea level is usually taken as zero. Look at the picture of the lighthouse. If the red fish is at -5 m (5 metres below sea level): Where is the yellow fish? Where is the green fish?</p>

This section reminds teachers to check pupils' understanding by asking questions such as 'Why?', 'What happens if ...', and checking that pupils can use the procedures or skills to solve a variety of problems.

This section contains examples of assessment questions, tasks and teaching activities that might support a teacher in assessing and evidencing progress of those pupils who have developed a sufficient grasp and depth of understanding so that learning is likely to be sustained over time.

This section contains examples of assessment questions, tasks and teaching activities that might support a teacher in assessing and evidencing progress of those pupils who have developed a stronger grasp and greater depth of understanding than that outlined in the first column.

Number and Place Value

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- count in multiples of 6, 7, 9, 25 and 1000
- order and compare numbers beyond 1000
- count backwards through 0 to include negative numbers
- round any number to the nearest 10, 100 or 1000

The Big Ideas

Imagining the position of numbers on a horizontal number line helps us to order them: the number to the right on a number line is the larger number. So 5 is greater than 4, as 5 is to the right of 4. But -4 is greater than -5 as -4 is to the right of -5 .

Rounding numbers in context may mean rounding up or down. Buying packets of ten cakes, we might round up to the nearest ten to make sure everyone gets a cake. Estimating the number of chairs in a room for a large number of people we might round down to estimate the number of chairs to make sure there are enough.

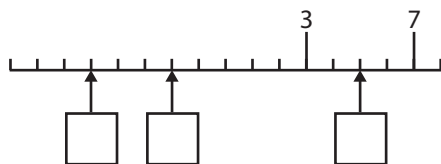
We can think of place value in additive terms: 456 is $400 + 50 + 6$, or in multiplicative terms: one hundred is ten times as large as ten.

Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?'; 'What happens if ...?'; and checking that pupils can use the procedures or skills to solve a variety of problems.

Mastery

Write the missing numbers in the boxes.



Mastery with Greater Depth

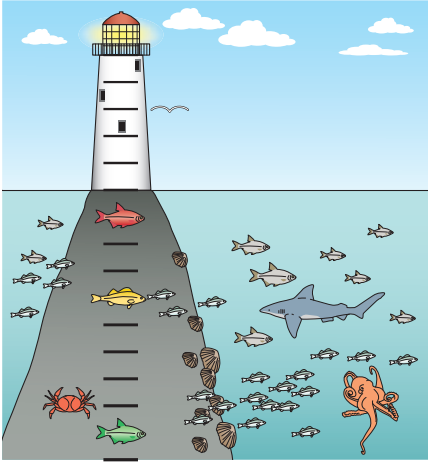
The sea level is usually taken as zero.

Look at the picture of the lighthouse.

If the red fish is at -5 m (5 metres below sea level):

Where is the yellow fish?

Where is the green fish?

Mastery	Mastery with Greater Depth
<p>What temperature is 20 degrees lower than 6 degrees Celsius?</p>	 <p>Can you draw a fish at -35 m?</p> <p>Can you draw a seagull at 20 m above sea level?</p> <p>What would the position of your fish and the seagull be if each of the intervals on the lighthouse represented 7 m?</p>
<p>Kiz has these numbers: 1330 1303 1033 1003 1030</p> <p>He writes them in order from smallest to largest. What is the fourth number he writes?</p> <p>Gemma counts on in 25s from 50. Circle the numbers that she will say:</p> <p>990 550 125 755 150</p>	<p>Here is a sequence of numbers: 20, 30, 40, 50</p> <p>What will the nineteenth number in the sequence be? What will the hundredth number in the sequence be?</p>
<p>Match 4600 to numbers with the same value.</p> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;">4600</div> <div style="flex-grow: 1;"> <p>460 tens</p> <p>460 hundreds</p> <p>46 hundreds</p> <p>4600 ones</p> <p>46 tens</p> </div> </div>	<p>How many different ways can you write 5510?</p> <p><i>Pupils should suggest answers such as:</i></p> <p>551 tens</p> <p>55 hundreds and 1 ten</p> <p>5 thousands and 510 ones</p>

Mastery

Using these 4 digits:

1

7

3

0

What is the smallest number you can make?

What is the largest number you can make?

Mastery with Greater Depth

5000 years ago Egyptians carved number symbols on their tombs:

 $| = 1$ $n = 10$ $@ = 100$

What is the value of these Egyptian numbers?

 $@nnlll$ $@nnlll + @nnlll$ $@@@nnlll + @@nnnnllllllll$

Addition and Subtraction

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- solve addition and subtraction two-step problems in context, deciding which operations and methods to use and why

The Big Ideas

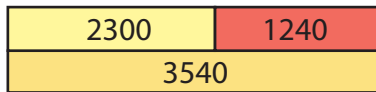
It helps to round numbers before carrying out a calculation to get a sense of the size of the answer. For example, $4786 - 2135$ is close to $5000 - 2000$, so the answer will be around 3000. Looking at the numbers in a calculation and their relationship to each other can help make calculating easier. For example, $3012 - 2996$. Noticing that the numbers are close to each other might mean this is more easily calculated by thinking about subtraction as difference.

Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?'; 'What happens if ...?'; and checking that pupils can use the procedures or skills to solve a variety of problems.

Mastery

Write down the four relationships you can see in the bar model.



$$\square + \square = \square$$

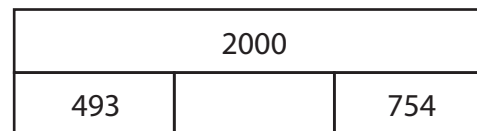
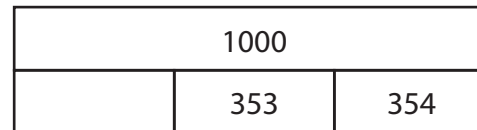
$$\square + \square = \square$$

$$\square - \square = \square$$

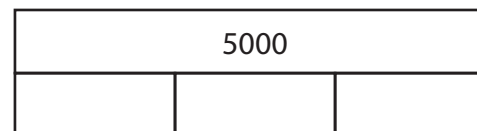
$$\square - \square = \square$$

Mastery with Greater Depth

Identify the missing numbers in these bar models. They are not drawn to scale.



Select your own numbers to make this bar model correct.



Mastery

Fill in the missing numbers.

$352 + \square = 480$

$70 + 99 + \square = 270$

$\square - 55 = 84$

$\square - 3000 = 600$

Mastery with Greater Depth

Fill in the missing digits.

$1\square 3 + 6\square = 200$

$1\square 5\square + 300 = 1557$

$5\square 28 - 44\square = 4788$

$\square\square\square 0 - 2468 = 5092$

What do you notice about the calculations below?
Can you find easy ways to calculate?

$5000 + 4000 =$

$5230 + 400 =$

$5023 + 28 =$

$4000 + 5000 =$

$4230 + 500 =$

$4023 + 28 =$

$3000 + 6000 =$

$3230 + 600 =$

$3023 + 28 =$

$2000 + 7000 =$

$2230 + 700 =$

$2023 + 28 =$

$1000 + 8000 =$

$1230 + 800 =$

$1023 + 48 =$

Find the missing numbers.
What do you notice?

Make 9999

$5000 + \square = 9999$

$4000 + \square = 9999$

$3000 + \square = 9999$

$2000 + \square = 9999$

$1000 + \square = 9999$

Make 9998

$5230 + \square = 9998$

$4230 + \square = 9998$

$3230 + \square = 9998$

$2230 + \square = 9998$

$1230 + \square = 9998$

Make 9990

$5023 + \square = 9990$

$4023 + \square = 9990$

$3023 + \square = 9990$

$2023 + \square = 9990$

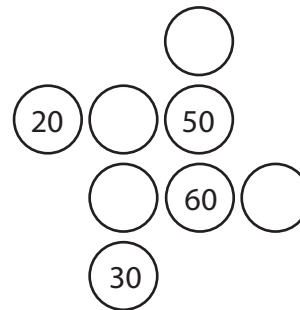
$1023 + \square = 9990$

Fill in the empty boxes to make the equations correct.

$\boxed{7}\boxed{}\boxed{1} + \boxed{}\boxed{3}\boxed{} = 999$

$\boxed{7}\boxed{}\boxed{1} + \boxed{}\boxed{3}\boxed{} = 1000$

Complete this diagram so that the three numbers in each row and column add up to 140.



Now create your own diagram with a total of 250.

Mastery	Mastery with Greater Depth
<p>Decide on a mental or written strategy for each of these calculations and perform them with fluency.</p> <ul style="list-style-type: none"> ■ $64 + 36$ ■ $640 + 360$ ■ $64 + 79 + 36$ ■ $378 + 562$ ■ $876 + 921$ ■ $999 + 999$ ■ $1447 + 2362$ ■ $1999 + 874$ 	<p>Write three calculations where you would use mental calculation strategies and three where you apply a column method.</p> <p>Explain the decision you made for each calculation.</p>
<p>Ali and Sarah calculate $420 + 221 + 280$ using different strategies.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p>This is Sarah's strategy:</p> $420 + 221 + 280$ $420 + 221 = 641$ $641 + 280 = 921$ <p>Answer = 921</p> </div> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p>This is Ali's strategy:</p> $420 + 221 + 280$ $420 + 280 = 700$ $700 + 221 = 921$ <p>Answer = 921</p> </div> </div> <p>Which do you prefer?</p> <p>Explain your reasoning.</p> <p>Now calculate $370 + 242 + 130$ using your preferred strategy.</p>	<p>Write $>$, $=$ or $<$ in each of the circles to make the number sentence correct.</p> $1023 + 24 + 24 \bigcirc 1023 + 48$ $1232 - 232 \bigcirc 1355 - 252$ $1237 - 68 + 32 \bigcirc 1242 - 69 + 31$ <p><i>Pupils should reason about the numbers and relationships, rather than calculate.</i></p>

Multiplication and Division

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1-digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

The Big Ideas

It is important for children not just to be able to chant their multiplication tables but to understand what the facts in them mean, to be able to use these facts to figure out others and to use them in problems.

It is also important for children to be able to link facts within the tables (e.g. $5 \times$ is half of $10 \times$).

They understand what multiplication means and see division as both grouping and sharing, and to see division as the inverse of multiplication.

The distributive law can be used to partition numbers in different ways to create equivalent calculations. For example, $4 \times 27 = 4 \times (25 + 2) = (4 \times 25) + (4 \times 2) = 108$.

Looking for equivalent calculations can make calculating easier. For example, 98×5 is equivalent to $98 \times 10 \div 2$ or to $(100 \times 5) - (2 \times 5)$. The array model can help show equivalences.

Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if ...?', and checking that pupils can use the procedures or skills to solve a variety of problems.

Mastery	Mastery with Greater Depth																								
<p>Use your knowledge of multiplication tables to complete these calculations.</p> <table border="1" data-bbox="98 264 887 545"> <tr> <td>$7 \times 6 =$</td> <td>$12 \times 6 =$</td> </tr> <tr> <td>$7 \times 2 \times 3 =$</td> <td>$13 \times 6 =$</td> </tr> <tr> <td>$8 \times 7 =$</td> <td>$12 \times 12 =$</td> </tr> <tr> <td>$2 \times 4 \times 7 =$</td> <td>$12 \times 13 =$</td> </tr> <tr> <td>$2 \times 2 \times 2 \times 7 =$</td> <td>$12 \times 0 =$</td> </tr> </table> <p>Which calculations have the same answer? Can you explain why?</p> <p><i>By the end of the year pupils should be fluent with all table facts up to 12×12 and also be able to apply these to calculate unknown facts, such as 12×13.</i></p>	$7 \times 6 =$	$12 \times 6 =$	$7 \times 2 \times 3 =$	$13 \times 6 =$	$8 \times 7 =$	$12 \times 12 =$	$2 \times 4 \times 7 =$	$12 \times 13 =$	$2 \times 2 \times 2 \times 7 =$	$12 \times 0 =$	<p>True or false?</p> <p>$7 \times 6 = 7 \times 3 \times 2$</p> <p>$7 \times 6 = 7 \times 3 + 3$</p> <p>Explain your reasoning.</p> <p>Can you write the number 30 as the product of 3 numbers?</p> <p>Can you do it in different ways?</p>														
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<p>What do you notice about the following calculations? Can you use one calculation to work out the answer to other calculations?</p> <table data-bbox="98 842 909 1123"> <tr> <td>$2 \times 3 =$</td> <td>$6 \times 7 =$</td> <td>$9 \times 8 =$</td> </tr> <tr> <td>$2 \times 30 =$</td> <td>$6 \times 70 =$</td> <td>$9 \times 80 =$</td> </tr> <tr> <td>$2 \times 300 =$</td> <td>$6 \times 700 =$</td> <td>$9 \times 800 =$</td> </tr> <tr> <td>$20 \times 3 =$</td> <td>$60 \times 7 =$</td> <td>$90 \times 8 =$</td> </tr> <tr> <td>$200 \times 3 =$</td> <td>$600 \times 7 =$</td> <td>$900 \times 8 =$</td> </tr> </table>	$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$	$2 \times 30 =$	$6 \times 70 =$	$9 \times 80 =$	$2 \times 300 =$	$6 \times 700 =$	$9 \times 800 =$	$20 \times 3 =$	$60 \times 7 =$	$90 \times 8 =$	$200 \times 3 =$	$600 \times 7 =$	$900 \times 8 =$	<p>Place one of these symbols in the circle to make the number sentence correct: $>$, $<$ or $=$.</p> <p>Explain your reasoning.</p> <table border="1" data-bbox="1115 928 1456 1193"> <tr> <td>8×50</td> <td><input type="radio"/></td> <td>50×8</td> </tr> <tr> <td>8×50</td> <td><input type="radio"/></td> <td>80×5</td> </tr> <tr> <td>300×3</td> <td><input type="radio"/></td> <td>5×200</td> </tr> </table>	8×50	<input type="radio"/>	50×8	8×50	<input type="radio"/>	80×5	300×3	<input type="radio"/>	5×200
$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$																							
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8×50	<input type="radio"/>	50×8																							
8×50	<input type="radio"/>	80×5																							
300×3	<input type="radio"/>	5×200																							

Mastery	Mastery with Greater Depth
<p>Three children calculated 7×6 in different ways. Identify each strategy and complete the calculations.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 30%;"> <p>Annie</p> $7 \times 6 = 7 \times 5 + \square$ $= \square$ </div> <div style="border: 1px solid black; padding: 5px; width: 30%;"> <p>Bertie</p> $7 \times 6 = 7 \times 7 - \square$ $= \square$ </div> <div style="border: 1px solid black; padding: 5px; width: 30%;"> <p>Cara used the commutative law</p> $7 \times 6 = \square \times \square$ $= \square$ </div> </div> <p>Now find the answer to 6×9 in three different ways.</p>	<p>Multiply a number by itself and then make one factor one more and the other one less. What happens to the product?</p> <p>E.g.</p> $4 \times 4 = 16$ $5 \times 3 = 15$ $6 \times 6 = 36$ $7 \times 5 = 35$ <p>What do you notice? Will this always happen?</p>
<p>Tom ate 9 grapes at the picnic. Sam ate 3 times as many grapes as Tom. How many grapes did they eat altogether?</p> <p><i>The bar model is a useful scaffold to develop fluency in this type of question.</i></p>	<p>Sally has 9 times as many football cards as Sam. Together they have 150 cards. How many more cards does Sally have than Sam?</p> <p><i>The bar model is a useful scaffold to develop fluency in this type of question.</i></p>

Fractions

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places

The Big Ideas



Fractions arise from solving problems, where the answer lies between two whole numbers.

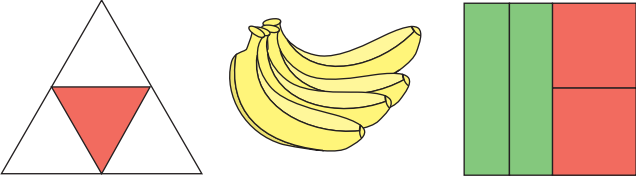
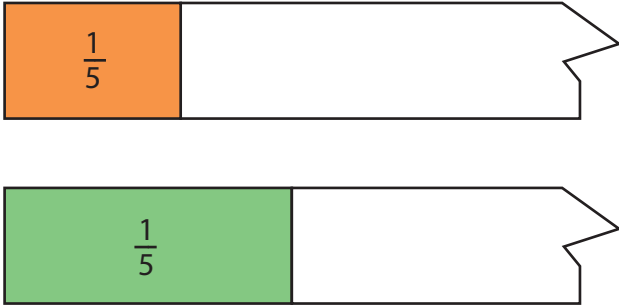
Fractions express a relationship between a whole and equal parts of a whole. Children should recognise this and speak in full sentences when answering a question involving fractions. For example, in response to the question *What fraction of the chocolate bar is shaded?* the pupil might say *Two sevenths of the whole chocolate bar is shaded.*


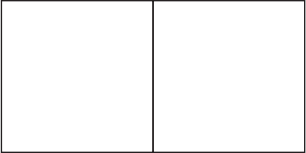
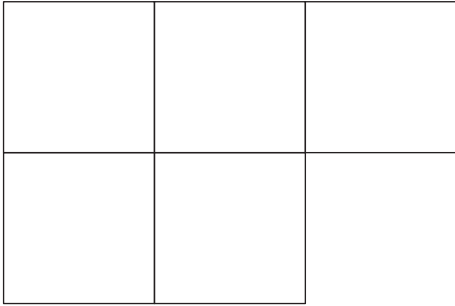
Equivalency in relation to fractions is important. Fractions that look very different in their symbolic notation can mean the same thing.

Mastery Check

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Mastery	Mastery with Greater Depth
Put these fractions on the number line: $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{6}$, $\frac{4}{9}$ 	Insert the symbol $>$, $<$ or $=$ to make each statement correct. $\frac{2}{5}$ of 5 \bigcirc $\frac{1}{4}$ of 4 $\frac{1}{7}$ of 7 \bigcirc $\frac{2}{7}$ of 14 $\frac{2}{3}$ of 9 \bigcirc $\frac{1}{3}$ of 18
Put these fractions on the number line: $\frac{4}{5}$, $\frac{7}{10}$, $\frac{5}{10}$, $\frac{2}{5}$ 	Make up three similar statements using $>$, $<$ or $=$.

Mastery	Mastery with Greater Depth
<p>What's the same? What's different?</p>  <p>Children should be able to express the ideas that:</p> <ul style="list-style-type: none"> ■ They are all divided into 4 equal parts. ■ Each part represents a quarter of the whole. ■ Each of the parts in the triangle are the same shape and area (congruent). ■ The shapes in the square are different but each has the same area (not congruent). ■ The bananas represent fractions of quantities. 	<p>Two paper strips are ripped. Identify which original paper strip is longer.</p> <p>Explain your answer.</p> 
<p>Draw diagrams to show two fractions that are equivalent to $\frac{2}{8}$.</p>	<p>How many ways can you express $\frac{2}{8}$ as a fraction?</p>
<p>8 girls share 6 bars of chocolate equally. 12 boys share 9 bars of chocolate equally. Who gets more chocolate to eat, each boy or each girl? How do you know?</p> <p>Draw a diagram to explain your reasoning.</p>	<p>8 girls share 6 bars of chocolate equally. 12 boys share 9 bars of chocolate equally.</p> <p>Clare says each girl got more to eat as there were fewer of them. Rob says each boy got more to eat as they had more chocolate to share.</p> <p>Explain why Clare and Rob are both wrong.</p>

Mastery	Mastery with Greater Depth
<p>Find:</p> <p>$\frac{1}{10}$ of 10</p> <p>$\frac{1}{10}$ of 20</p> <p>$\frac{1}{10}$ of 30</p> <p>$\frac{1}{10}$ of 40</p> <p>$\frac{1}{10}$ of 50</p> <p>What do you notice?</p>	<p>Captain Conjecture says,</p> <p>'To find a tenth of a number I divide by 10 and to find a fifth of a number I divide by 5.'</p> <p>Do you agree?</p> <p>Explain your reasoning.</p> 
<p>If the picture represents $\frac{2}{12}$ of a rectangle, draw a picture of the whole rectangle.</p> <p>Can you draw it in two different ways?</p> 	<p>If the picture represents $\frac{1}{3}$ of a shape, draw the whole shape.</p> 
<p>True or false?</p> <p>$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$</p> <p>$\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$</p> <p>$\frac{1}{5} + \frac{2}{5} = \frac{6}{10}$</p> <p>Explain your reasoning.</p>	<p>Peter wrote down two fractions. He subtracted the smaller fraction from the larger and got $\frac{1}{8}$ as the answer.</p> <p>Write down two fractions that Peter could have subtracted.</p> <p>Can you find another pair?</p>

Mastery

Match each fraction to its decimal equivalent.

$$\frac{1}{2} \quad \frac{4}{10} \quad \frac{3}{4} \quad \frac{1}{4}$$

$$0.25 \quad 0.75 \quad 0.4 \quad 0.5$$

Circle the equivalent fraction to 0.25.

$$\frac{2}{5} \quad \frac{5}{2} \quad \frac{25}{100} \quad \frac{100}{25}$$

Round to the nearest whole number.

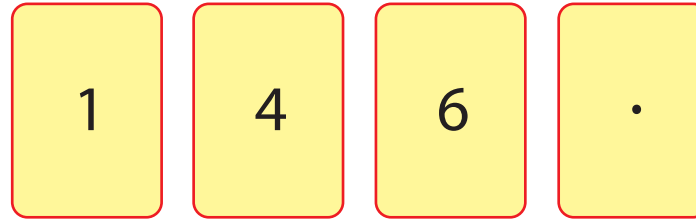
$$8\frac{3}{8} \quad 8.38 \quad 8.83$$

A soup recipe uses $\frac{3}{4}$ as many onions as carrots. Jo is making the soup and has 8 carrots.

How many onions does Jo use?

Mastery with Greater Depth

Using these cards can you make a number between 4.1 and 4.61?



What is the smallest number you can make using all four cards?

What is the largest number you can make using all four cards?

A soup recipe uses $\frac{3}{4}$ as many onions as carrots.

Complete the table below.

Carrots	Onions
1	
2	
3	
4	
5	
6	

Explain how you worked out the number of onions. Did you use the same method each time?

Measurement

Selected National Curriculum Programme of Study Statements

Pupils are taught to

- convert between different units of measure [for example, kilometre to metre; hour to minute]
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- estimate, compare and calculate different measures, including money in pounds and pence

The Big Idea

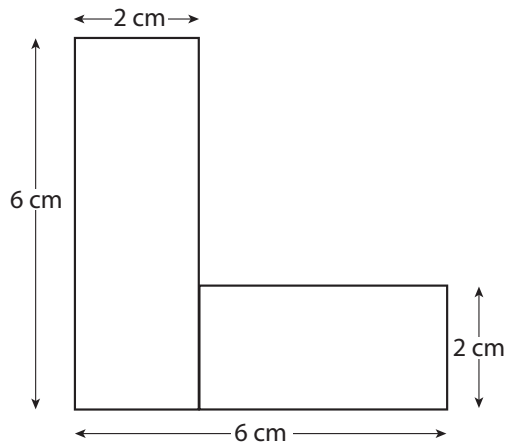
The smaller the unit, the greater the number of units needed to measure (that is, there is an inverse relationship between size of unit and measure).

Mastery Check

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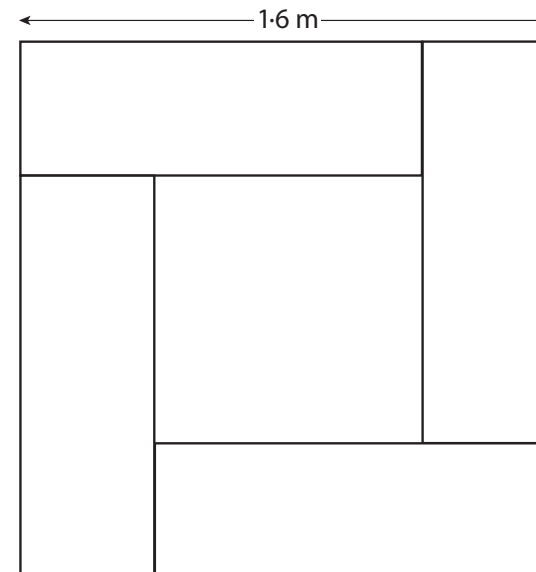
Mastery

The shape below is made from two rectangles.
Identify the perimeter of each of the two rectangles.
How many 1 cm squares would fit into the smaller rectangle?
How many more squares fit into the larger rectangle?



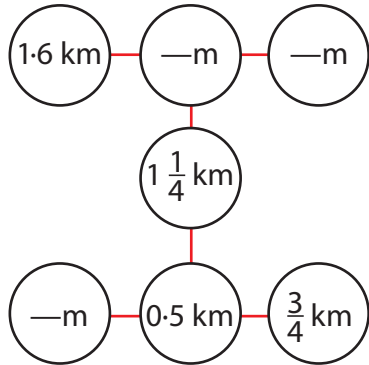
Mastery with Greater Depth

The rectangular tiles here are three times as long as they are wide.
What is the perimeter of the centre square?



Mastery

Complete the missing measures so that each line of three gives a total distance of 2 km.



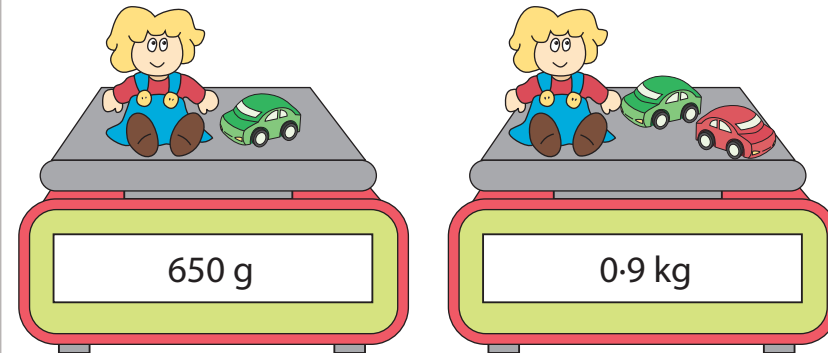
An empty box weighs 0.5 kg. Ivy puts 10 toy bricks inside it and the box now weighs 2 kg.

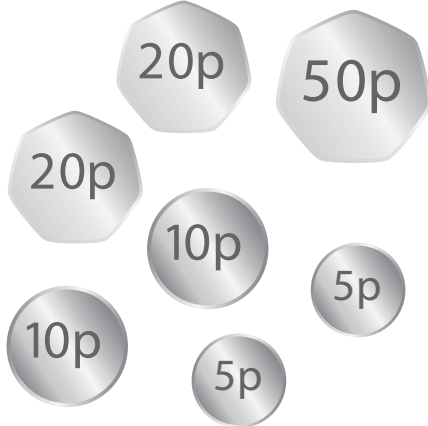
How much does each brick weigh?

Mastery with Greater Depth

In total Sam and Tom together cycle a distance of 120 km. Sam cycles twice the distance that Tom cycles. How far does Sam cycle?

How much does the car weigh in grams?
How much does the doll weigh in grams?



Mastery	Mastery with Greater Depth					
<p>Which would you rather have, $3 \times 50\text{p}$ coins or $7 \times 20\text{p}$ coins?</p> <p>Explain your reasoning.</p>	<p>Sid and Sam share some money. Sid gets twice as much as Sam. Tick the coins which Sid might take.</p>  <p>Is there more than one way of sharing the coins?</p>					
<p>Put these amounts in order starting with the largest.</p> <ul style="list-style-type: none"> ■ Half of 3 litres ■ Quarter of 2 litres ■ 300 ml <p>Explain your thinking.</p>	<p>Fill in the missing boxes so that the amounts are in order from smallest to greatest.</p> <table border="1" data-bbox="1115 919 2002 1050"> <tr> <td style="text-align: center;">$\frac{1}{2}$ a litre</td> <td style="text-align: center;"><input type="text"/> millilitres</td> <td style="text-align: center;">$\frac{1}{3}$ of 2 litres</td> <td style="text-align: center;"><input type="text"/> millilitres</td> <td style="text-align: center;">$\frac{1}{4}$ of 3 litres</td> </tr> </table>	$\frac{1}{2}$ a litre	<input type="text"/> millilitres	$\frac{1}{3}$ of 2 litres	<input type="text"/> millilitres	$\frac{1}{4}$ of 3 litres
$\frac{1}{2}$ a litre	<input type="text"/> millilitres	$\frac{1}{3}$ of 2 litres	<input type="text"/> millilitres	$\frac{1}{4}$ of 3 litres		

Geometry

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
- identify acute and obtuse angles and compare and order angles up to two right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry

The Big Ideas

During this year, pupils increase the range of 2-D and 3-D shapes that they are familiar with. They know the correct names for these shapes, but, more importantly, they are able to say why certain shapes are what they are by referring to their properties, including lengths of sides, size of angles and number of lines of symmetry.

The naming of shapes sometimes focuses on angle properties (e.g. a rectangle is right-angled), and sometimes on properties of sides (e.g. an equilateral triangle is an equal sided triangle).

Shapes can belong to more than one classification. For example, a square is a rectangle, a parallelogram, a rhombus and a quadrilateral.

Mastery Check

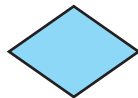
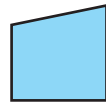
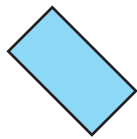
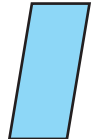
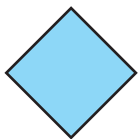
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Mastery

Below are five quadrilaterals: a rectangle, a rhombus, a square, a parallelogram and an unnamed quadrilateral.

Write the names of each of the quadrilaterals.

Draw lines from each shape to match the properties described in the boxes below.



All sides equal

Has an acute angle

Opposite sides are of equal length

All 4 angles are equal

Has an obtuse angle

Mastery with Greater Depth

Captain Conjecture says that a rectangle is a regular shape because it has four right angles.

Do you agree?

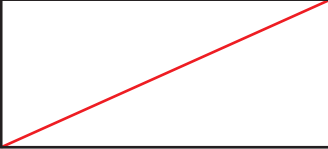
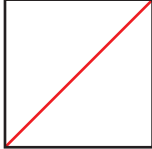
Explain your reasoning.

Captain Conjecture says that a quadrilateral can sometimes only have three right angles.

Do you agree?

Explain your reasoning.



Mastery	Mastery with Greater Depth
<p>Draw some 2-D shapes that have:</p> <ul style="list-style-type: none">■ no lines of symmetry■ 1 line of symmetry■ 2 lines of symmetry.	<p>Tom says, 'In each of these shapes the red line is a line of symmetry.' Do you agree?</p> <p>Explain your reasoning.</p> <div style="display: flex; justify-content: space-around; align-items: center;"></div>

Statistics

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs
- solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs

The Big Ideas

In mathematics the focus is on numerical data. These can be discrete or continuous. Discrete data are counted and have fixed values, for example the number of children who chose red as their favourite colour (this has to be a whole number and cannot be anything in between). Continuous data are measured, for example at what time did each child finish the race? (Theoretically this could be any time: 67.3 seconds, 67.33 seconds or 67.333 seconds, depending on the degree of accuracy that is applied.) Continuous data are best represented with a line graph where every point on the line has a potential value.

Mastery Check

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Mastery

Check that children can answer questions about data presented in different ways:

- *Are they able to make connections when looking at the same data presented differently?*
- *Can they answer questions about the data using inference and deduction or only direct retrieval?*
- *Are they able to present data in different ways?*
- *Do they label axes correctly?*
- *Do they understand the scale and do they use an appropriate scale when presenting data?*

Mastery with Greater Depth

Children hypothesise beyond the data that are presented, asking and answering questions such as 'What would happen if..?'

Mastery

Here is a table of the average temperature for each month of last year:

Month	1	2	3	4	5	6	7	8	9	10	11	12
Average Temp (°C)	6	7	10	12	16	18	21	22	18	14	10	7

Answer the questions below and explain your reasoning:

- On average what was the hottest month of the year?
- In which months was the average temperature below 10°C?
- In which months would you choose to go outside without your coat on?

Choose another way to represent the data.

Mastery with Greater Depth

Here is a table of the average temperature for each month of last year:

Month	1	2	3	4	5	6	7	8	9	10	11	12
Average Temp (°C)	6	7	10	12	16	18	21	22	18	14	10	7

Write the word 'true', 'false' or 'unknown' next to each statement, giving an explanation for each response.

- I would need to wear my coat outside in January.
- The hottest day of the year was in August.
- A temperature of -2 was recorded in January.

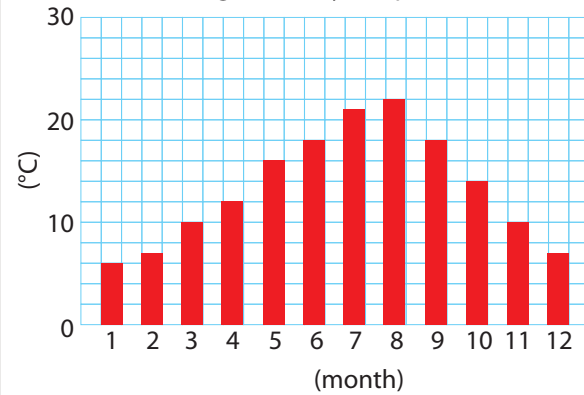
Choose two other ways to represent the data.

Mastery

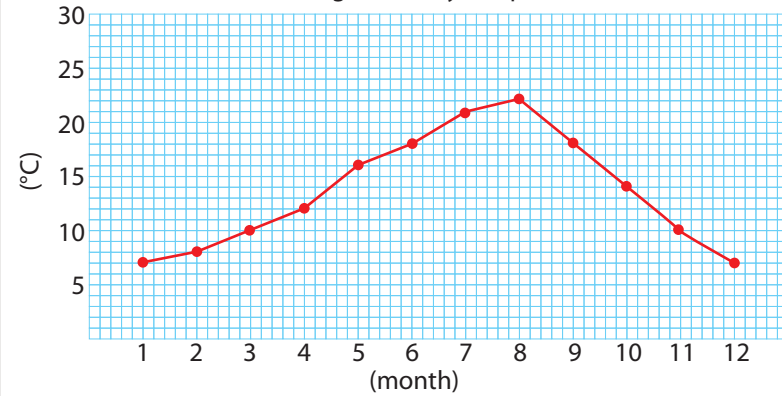
These two graphs represent the same data.

What's the same? What's different?

Average monthly temperature



Average monthly temperature



Which graph is better?

Explain your reasoning.

Mastery with Greater Depth

Make up a story that fits the graph.

